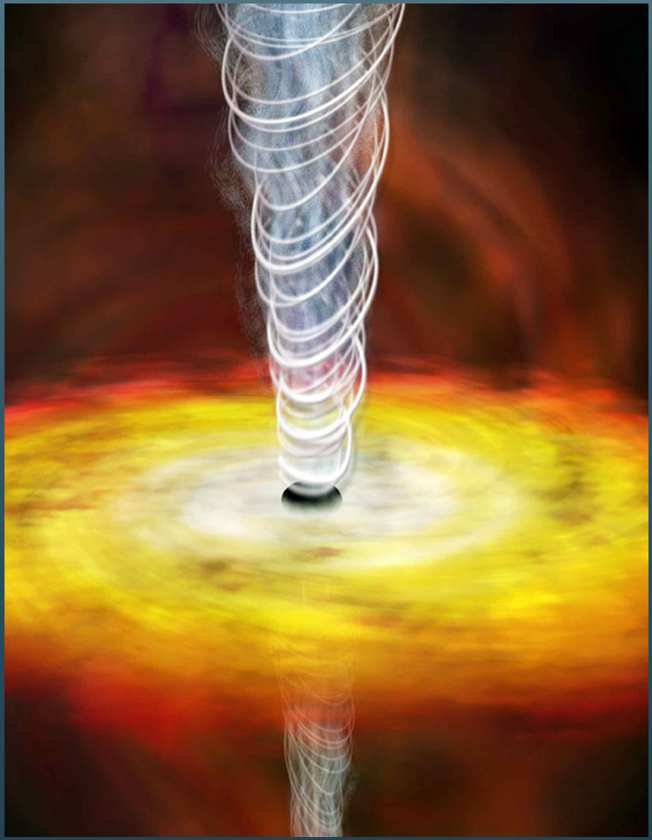
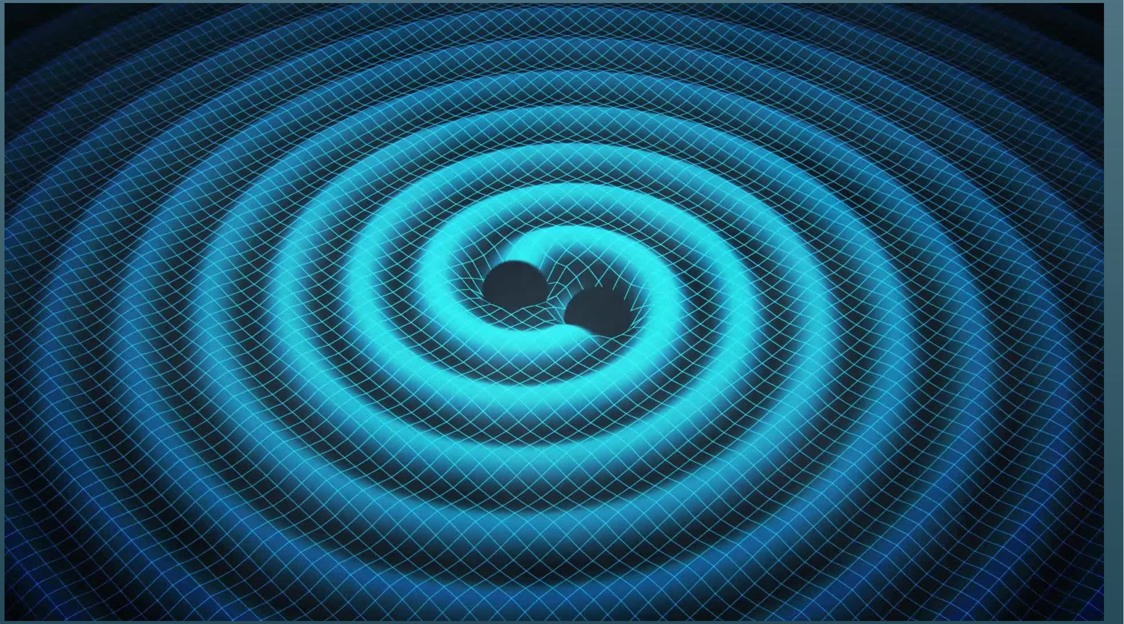
An Overview of IllinoisGRMHD

Zachariah Etienne





Difficulties in solving GRMHD equations numerically

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Context: model strong, rapidly-changing gravitational-field astrophysical phenomena (e.g., short GRBs, BBHs in disks)

- **Difficulty #1:**
  - Disparate length/time scales
    - BH horizon scales/NS radii
      - must resolve strong-field region
    - Gravitational wavezone
      - GW lengthscale ~ 10^3 dx_{strong}
- Solutions:
  - Use non-uniform numerical grids E.g., AMR
  - Develop codes for massively parallel HPC simulations
  - Sometimes wait months for results! (Moore's Law helps...)

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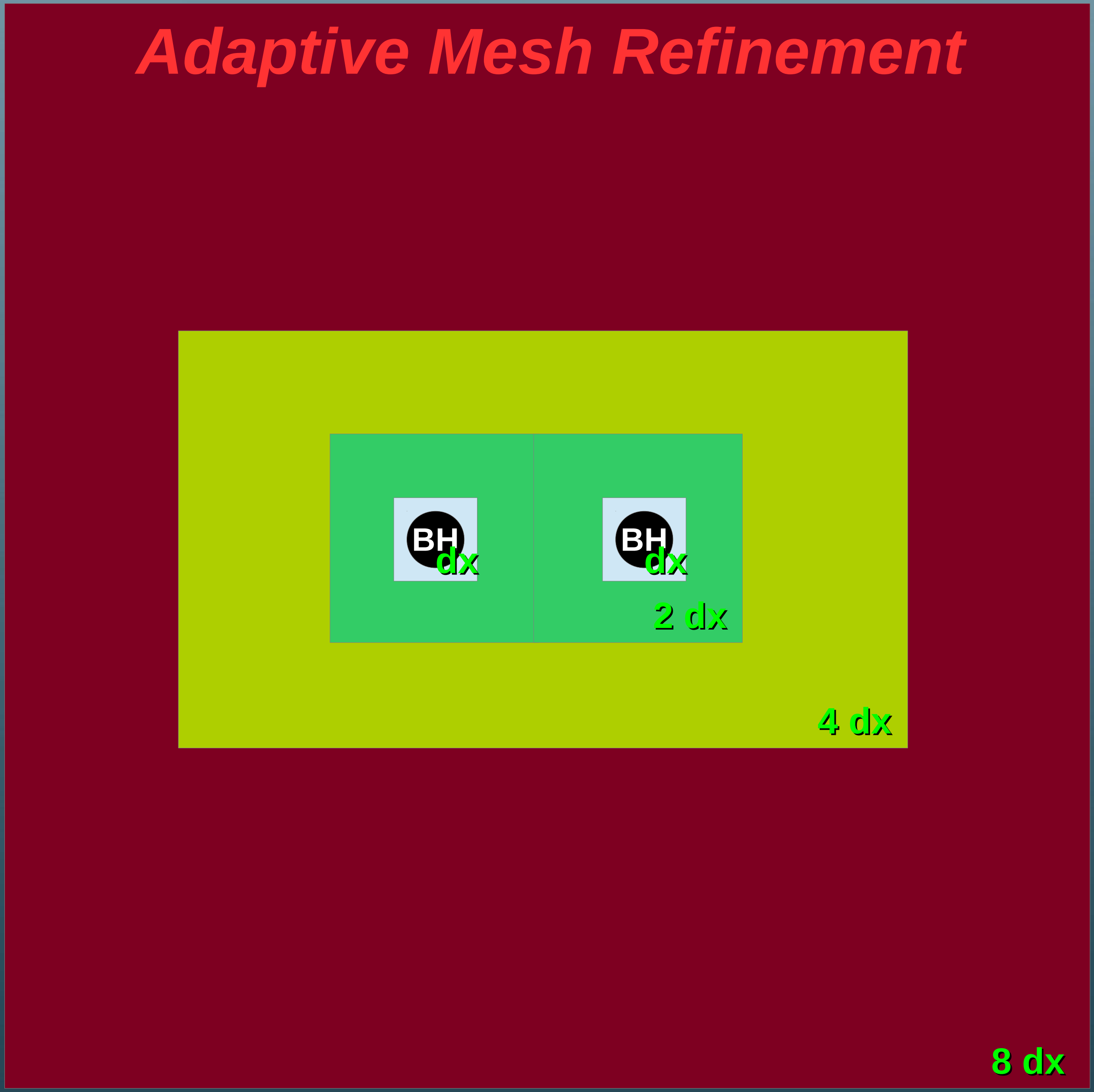

16 dx, etc

Difficulties in solving GRMHD equations numerically

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- **Difficulty #2:** $\partial_t \tilde{B}^i + \partial_j (v^j \tilde{B}^i - v^i \tilde{B}^j) = 0$ $\partial_j \tilde{B}^j = 0$
  - div B = 0 constraint: Take div of induction equation → if div B=0 initially, stays zero... to truncation error.
  - Truncation errors may follow random or directed walk, growing over time → monopole problem gets worse and worse!
- Popular solutions:
  - Special finite differencing performed on induction equation ("flux-CT" or "flux-CD" scheme)
    - → Div B remains at initial value to roundoff error for all time
  - Hyperbolic divergence cleaning (adds add'l MHD modes that sweep away and damp B-field divergence)

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- **Difficulty #3:**
  - Div B = 0 constraint + AMR.
    - AMR requires quantities to be interpolated at grid boundaries.
    - Interpolation errors on B-fields → monopoles.
    - How to interpolate B-fields at AMR boundaries?
- Solutions:
  - Hyperbolic divergence cleaning (adds add'l MHD modes that sweep away and damp B-field divergence)
  - Special interpolation operators (E.g., scheme of Balsara)
  - Evolve vector potential **A** instead, any interpolation scheme works!

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- **Difficulty #4:**
  - Strong hydrodynamic & MHD shocks
- Solution: High-resolution shock-capturing scheme
  - Requires that GRMHD equations be written in conservative form (Lec1!)
- Algorithmic Ingredients:
  - Reconstruction scheme: interpolates MHD quantities to cell interfaces (between gridpoints), minimizing Gibbs oscillations
  - Riemann Solver: Approximately solves the Rankine-Hugenoit conditions at shock interfaces
  - Conservative-to-Primitive solver: converts set of conserved evolved variables into “primitive” variables (e.g., density, velocity, pressure)
    - Nonlinear relation between Conservative & Primitive variables
      - → Need fast, robust root finder! (E.g., multi-D Newton-Raphson)

Difficulties in solving GRMHD equations numerically

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- **Difficulty #5:**
  - MHD flows into black holes
- Solutions:
  - Excise GRMHD data deep inside black hole
    - Problem: Can be fickle!
      - Control theory problem
      - Gauge characteristic modes can be superluminal, so if B significantly contributes to Tmunu and is sharp, can get code crashes!
      - Can result in nasty B-field build-up that percolates to the horizon edge, and then outside → code crash!
  - Check conservative variables *prior* to converting to primitive variables, move to physically valid range.
    - Appears to be quite robust and hands-free, so long as fluid densities do not blow up inside the horizon. (Easy fix via density ceiling.)

Difficulties in solving GRMHD equations numerically

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- **Difficulty #6:**
  - Regions where B-field dominates dynamics: P_B / P_gas >> 1
    - Truncation errors from evolving B can be larger than magnitude of P, rho!
    - Usually a problem in very low-density regions
- Solutions:
    - Density floor: sets an effective ceiling on P_B / P_gas
    - Fix P_B/P_gas to some large but trustworthy value in low-density regions

$$T^{\mu\nu} = (\rho_0 h + b^2) u^\mu u^\nu + \left(P + \frac{b^2}{2}\right) g^{\mu\nu} - b^\mu b^\nu$$

$$\partial_t \tilde{\tau} + \partial_i (\alpha^2 \sqrt{\gamma} T^{0i} - \rho_* v^i) = s \ ,$$

$$\partial_t \tilde{S}_i + \partial_j (\alpha \sqrt{\gamma} T^j{}_i) = \frac{1}{2} \alpha \sqrt{\gamma} T^{\alpha\beta} g_{\alpha\beta,i} \ ,$$

$$b^\mu = B^\mu_{(u)} / \sqrt{4\pi} \text{ and } b^2 = b^\mu b_\mu$$

Open Source Numerical Relativity: Einstein Toolkit Rules!

- Einstein Toolkit: ***http://einsteintoolkit.org***
  - Collection of open-source codes for NR, includes many of these:
- Kranc ***http://kranccode.org***
  - Mathematica-based code generation software
    - Handles complex tensor expressions, e.g., BSSN(OK)
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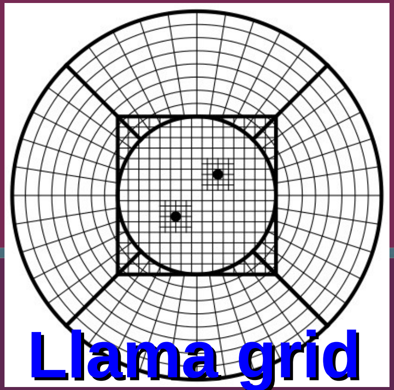
- Cactus/Carpet/Llama ***http://[cactus,carpet,llama]code.org***
  - Grid codes with excellent scalability



Pollney, Reisswig, Schnetter, Dorband, Diener: PRD 83 044045 (2011)

- GRHydro ***http://einsteintoolkit.org***
  - GRMHD HRSC code, many features, evolves B-field+div cleaning
- IllinoisGRMHD ***http://tinyurl.com/ilgrmhd***
  - Based on/identical results to decade-old GRMHD code of Illinois group
  - Designed for AMR grids & robustness (e.g., flows into BHs, no excision)
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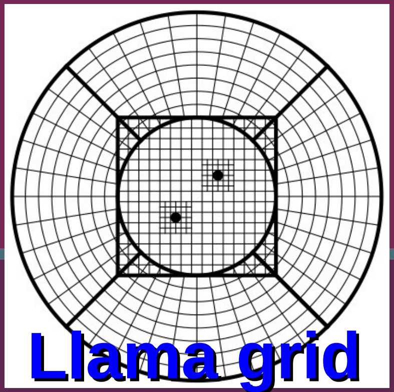

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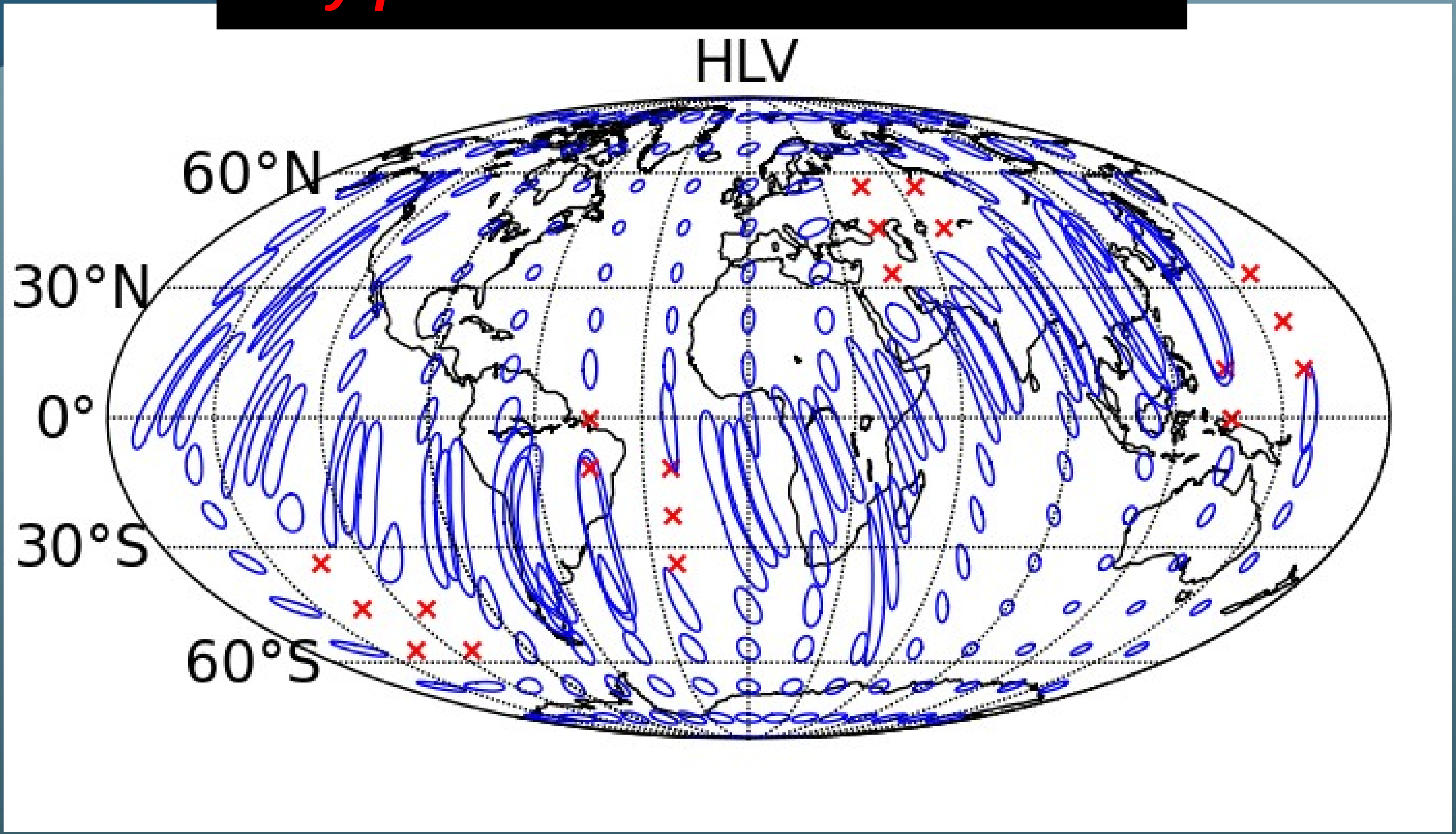
Very poor GW source localization



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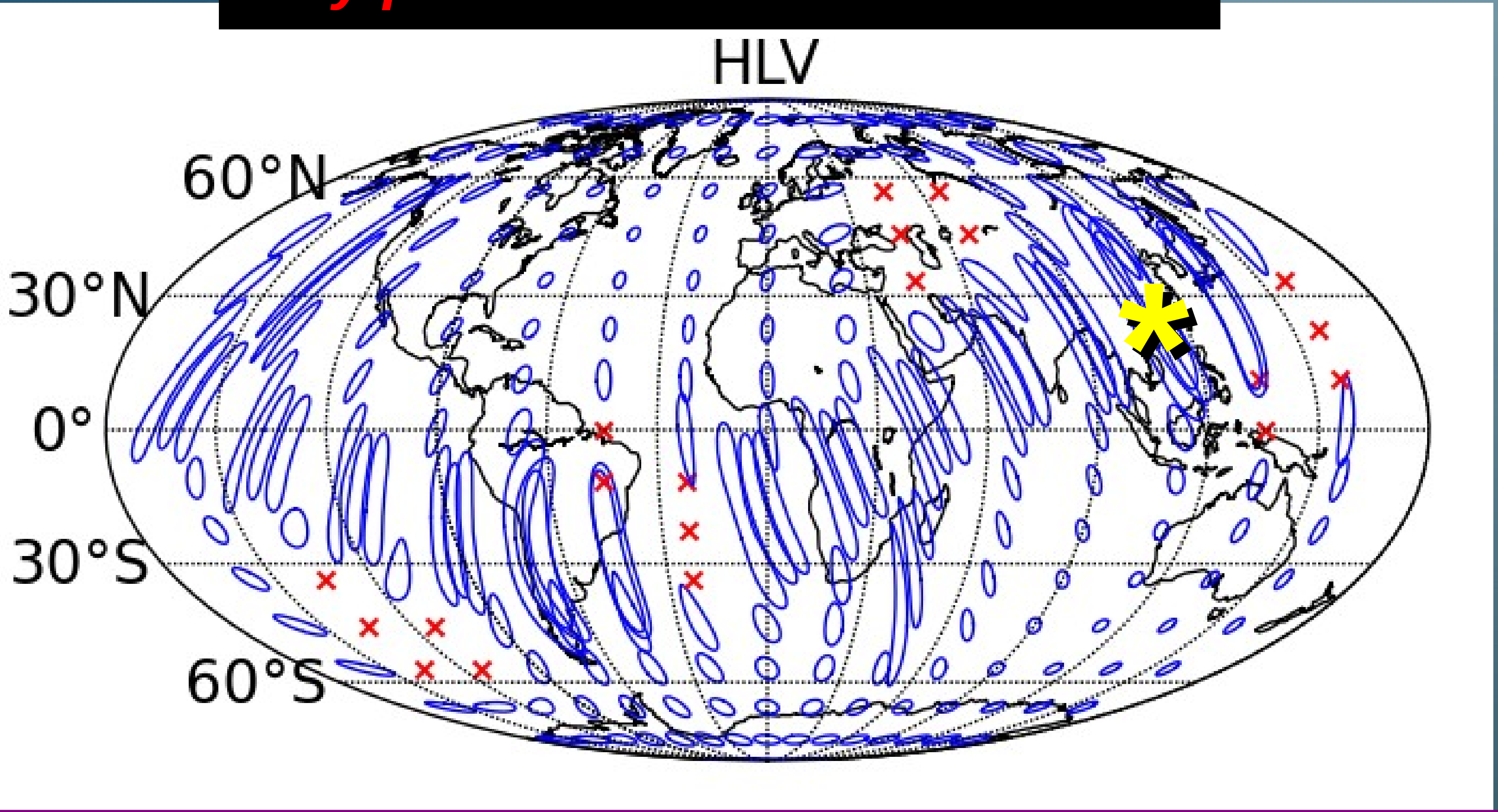

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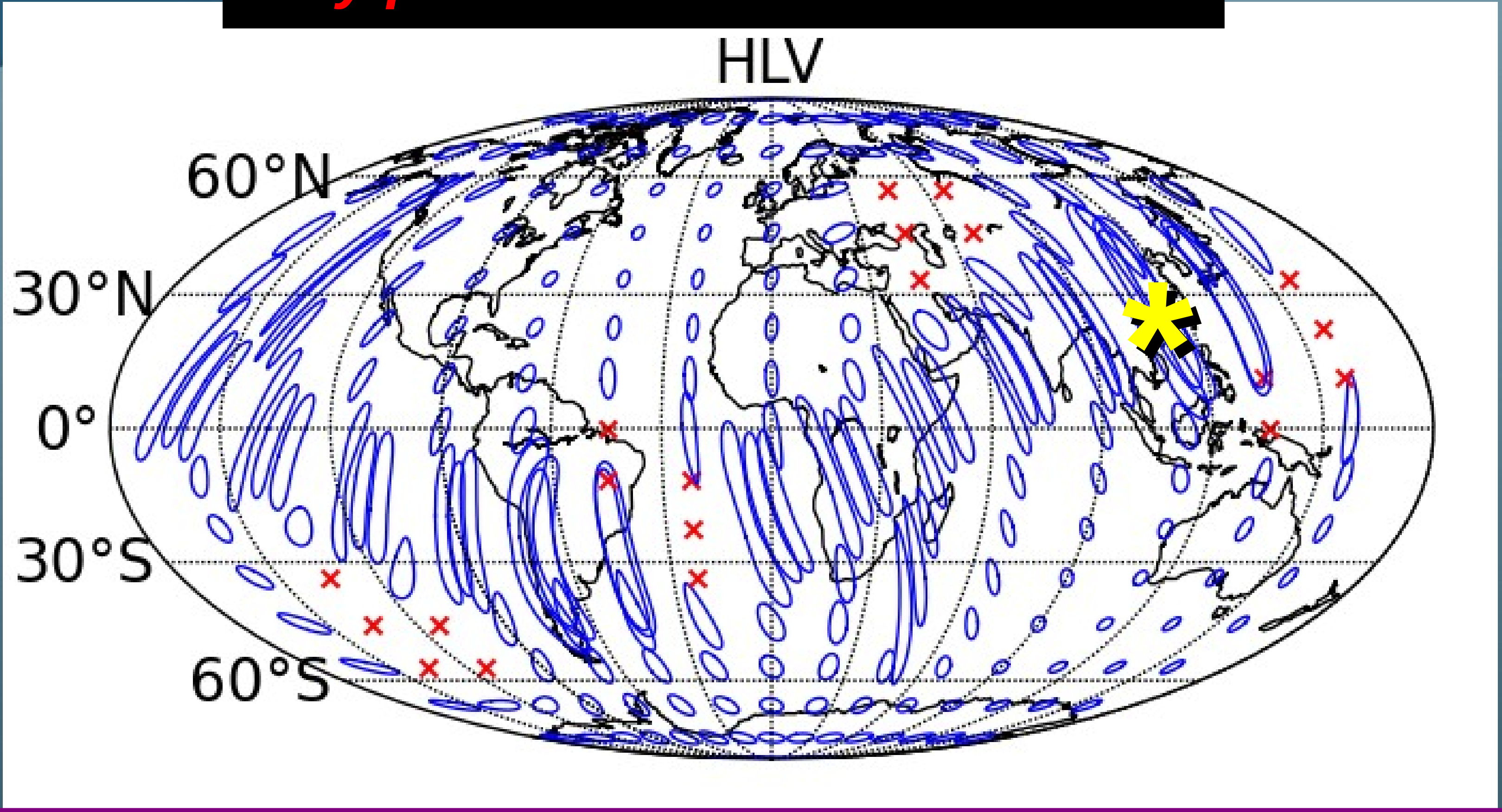

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- ***What sort of counterpart to look for?***
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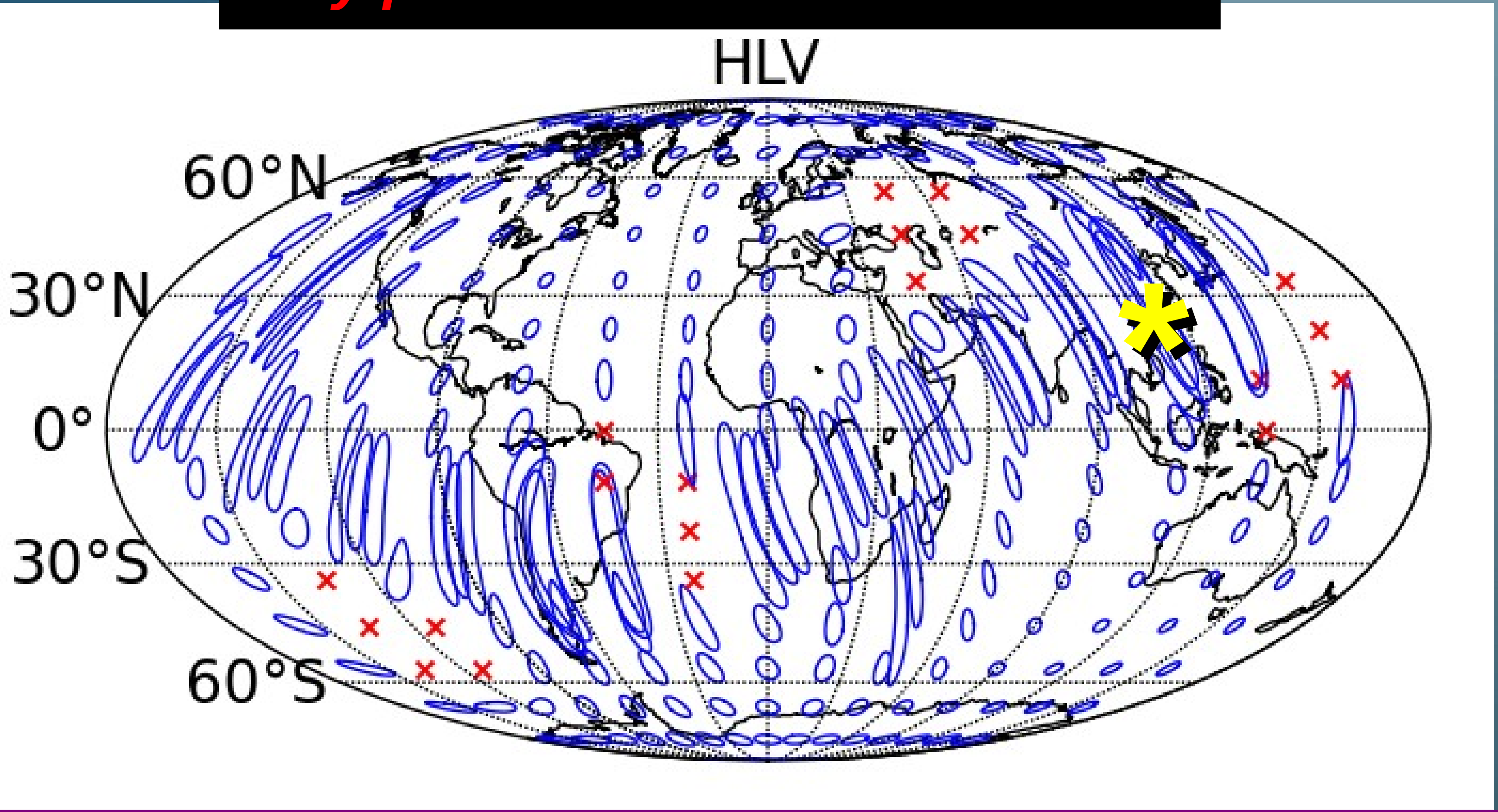

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Need dyn spacetime GRMHD models!

The Need for GRMHD Modeling

- GRMHD phenomena in dynamical spacetimes
  - Yield detectable EM signatures
    - Important for GW source localization
    - May lead to directed GW searches
    - Even *without* coincident GWs
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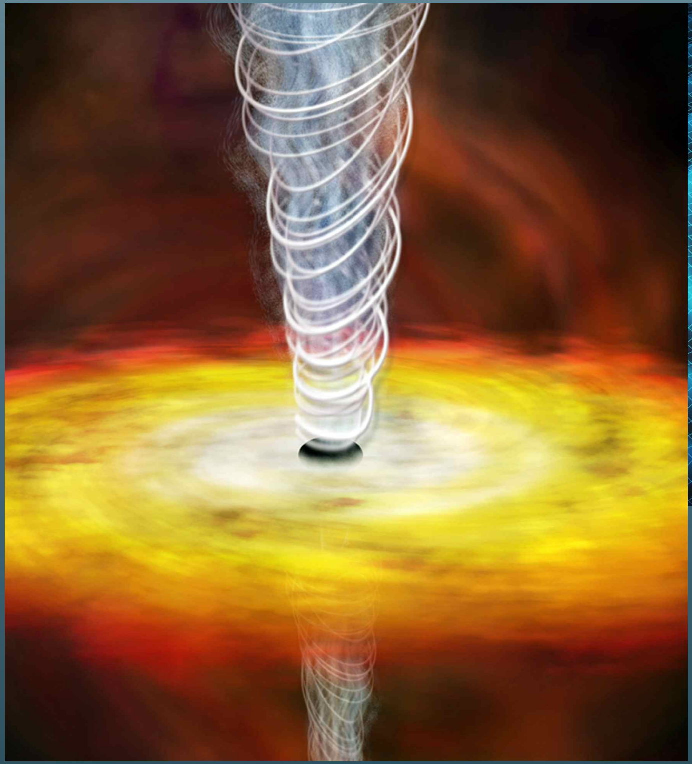
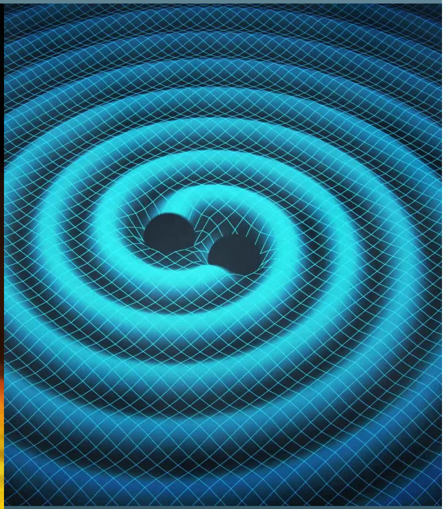
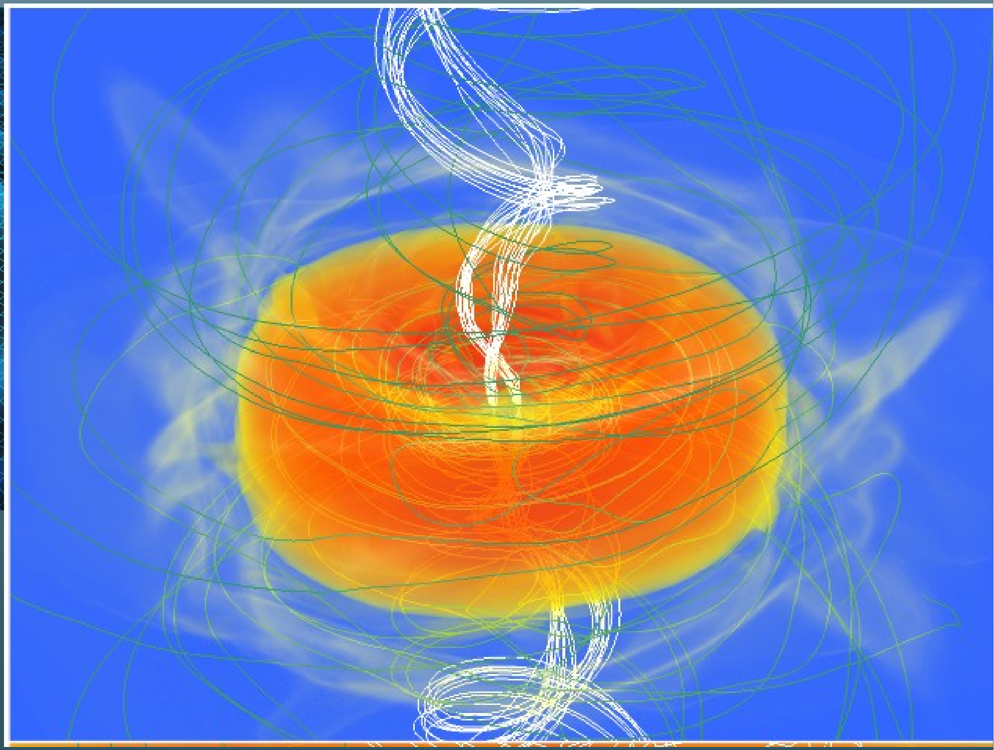
- Problems:
  - What to look for?
  - How to gain insights from EM observations?
- Strategy:
  - Model GRMHD phenomena in dyn spacetimes
    - IllinoisGRMHD does this!

IllinoisGRMHD: A User-Friendly GRMHD Code for Dynamical Spacetimes







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- Same robustness
- Well documented
- Easy mastery

IllinoisGRMHD

IllinoisGRMHD: The Need for Change

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  - The Good:
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    - Poorly commented/documented
  - The Ugly

    - Variety of coding styles, in FORTRAN90 (drivers) and C++ (low-level OpenMP'ed routines)
    - ~90% of original features unused & unmaintained
      - ~70k lines:
        - Mostly dead code, full of if() statements
        - Extremely difficult to navigate, even for experts
        - Function duplication throughout – not extensible

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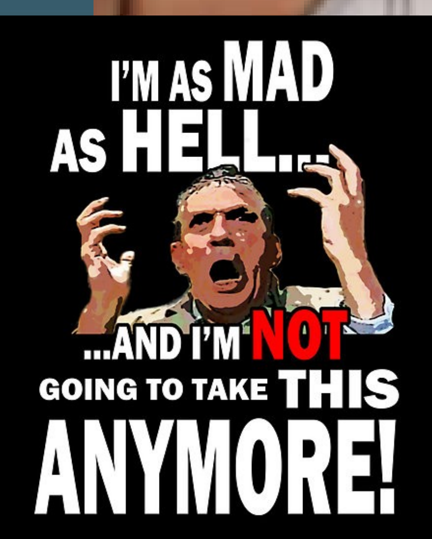

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IllinoisGRMHD: Future Plans

- Immediate: Incorporation into the next ET Release
  - All items on IllinoisGRMHD's TODO list (courtesy R. Haas) preventing inclusion into ET have been addressed
  - Good to go. Please take a look!
- Upcoming year:
  - SymBase support, arbitrary EOS support
  - Make IllinoisGRMHD, other GRMHD codes ***even more robust***
    - WhiskyMHD dyn spacetime GRMHD (B. Giacomazzo)
    - HARM3D fixed spacetime GRMHD (J. McKinney)

End

Start of Extra Slides

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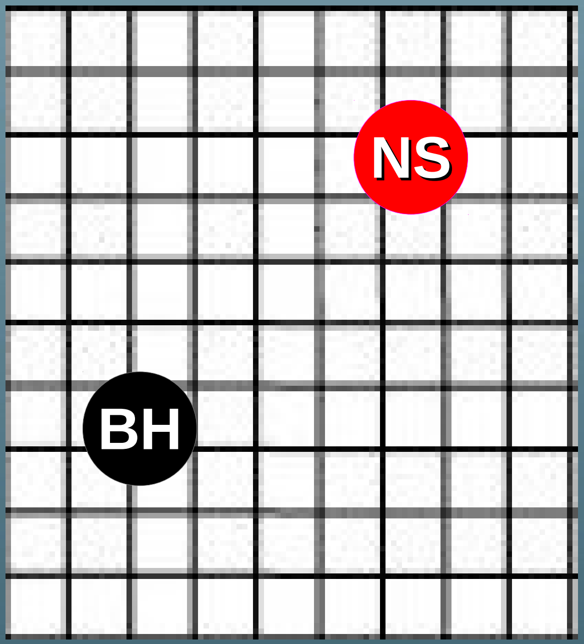

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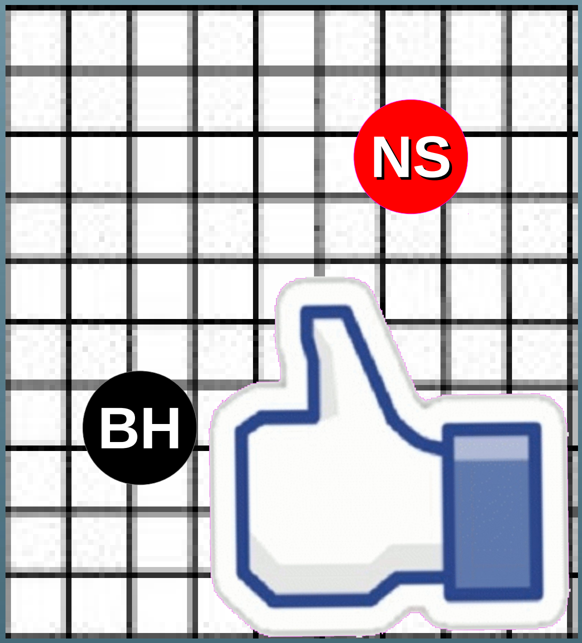

Gauge invariant!

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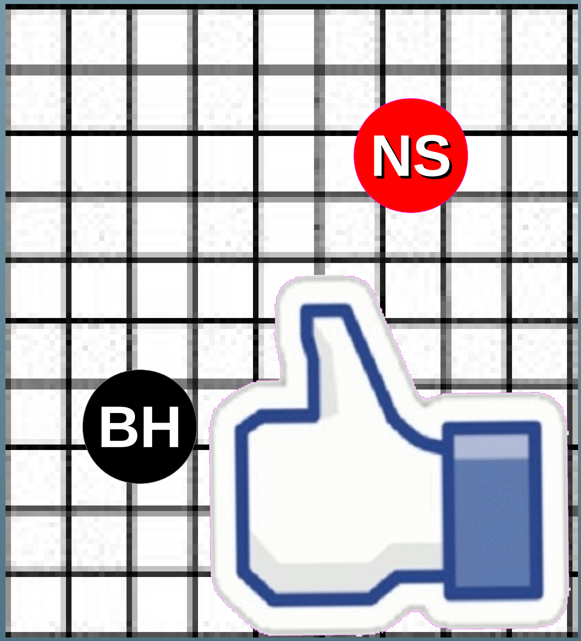

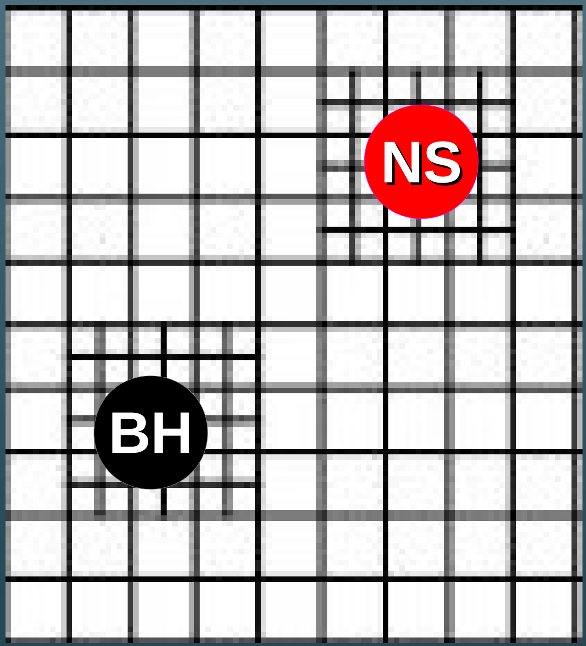

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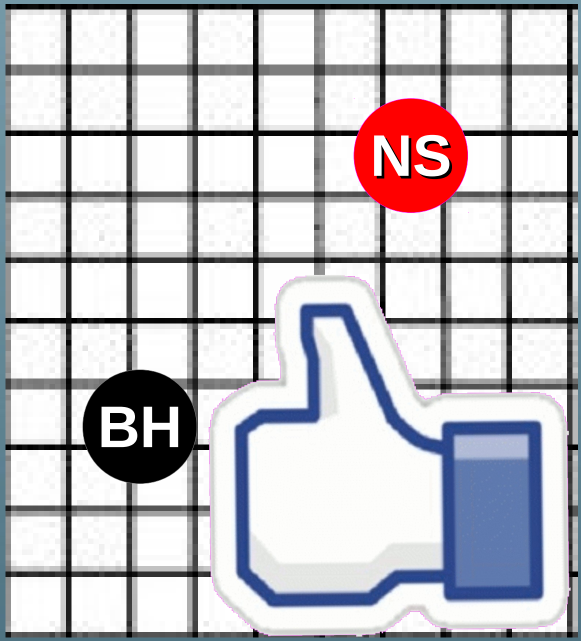

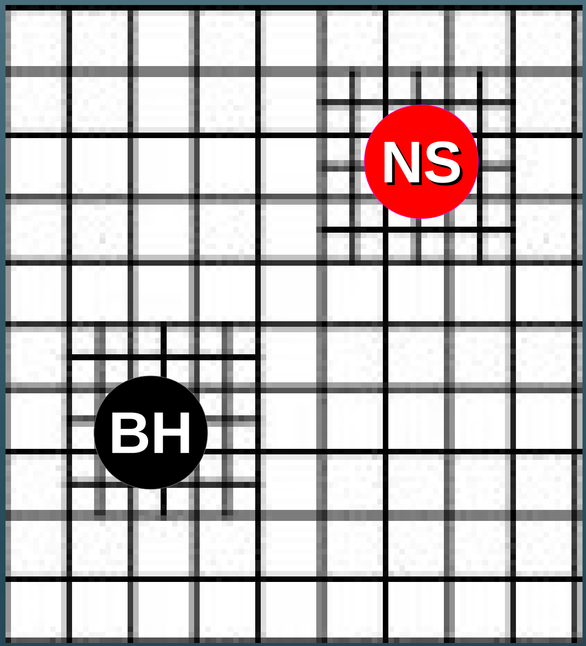

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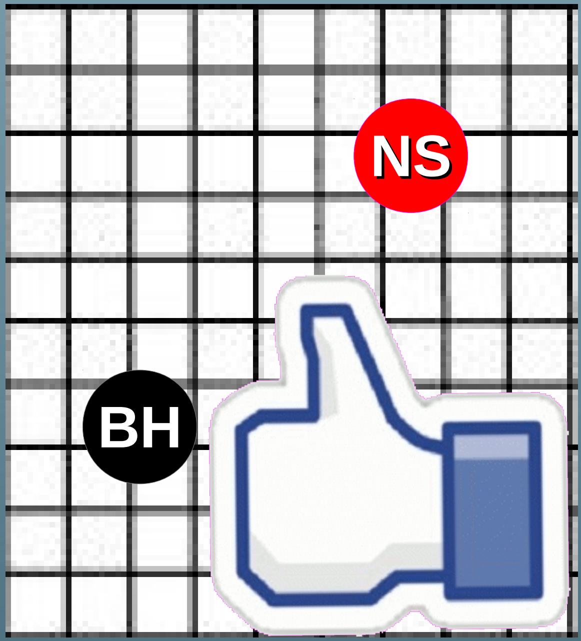

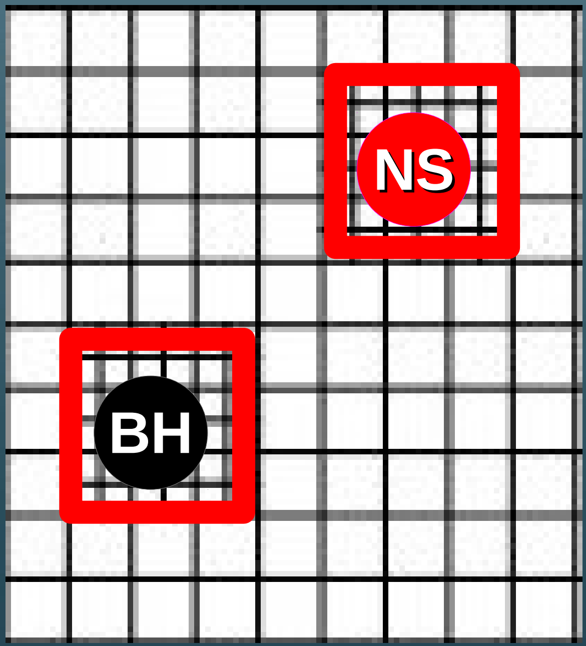

Must be careful!

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$$\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) \quad , \quad \nabla \rightarrow \mathbf{k} \ (\text{wavevector with } |\mathbf{k}|=1)$$

$$\Rightarrow \quad \partial_t \mathbf{A} = \mathbf{M}\mathbf{A} \quad , \quad M_{ij} = k_i \mathrm{v}_j - (\mathbf{v}\cdot\mathbf{k})\delta_{ij}$$

Eigenvalues of $\mathbf{M}$: $\boxed{\lambda_1 = 0,} \quad \lambda_2 = \lambda_3 = \mathbf{v}\cdot\mathbf{k}$

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Error build up on AMR grids

$$\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) \quad , \quad \nabla \to \mathbf{k} \text{ (wavevector with } |\mathbf{k}|=1)$$

$$\Rightarrow \quad \partial_t \mathbf{A} = \mathbf{M}\mathbf{A} \quad , \quad M_{ij} = k_i \mathrm{v}_j - (\mathbf{v} \cdot \mathbf{k})\delta_{ij}$$

Eigenvalues of $\mathbf{M}$: $\boxed{\lambda_1 = 0,}\quad \lambda_2 = \lambda_3 = \mathbf{v} \cdot \mathbf{k}$

Zero-speed mode!

$$A^\mu = (\Phi, \mathbf{A})$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\partial_t \mathbf{A} = \mathbf{v} \times \mathbf{B} - \nabla \Phi$$

Lorenz gauge condition:

$$\nabla_\mu A^\mu = 0 \quad \Rightarrow \quad \partial_t \Phi + \nabla \cdot \mathbf{A} = 0$$

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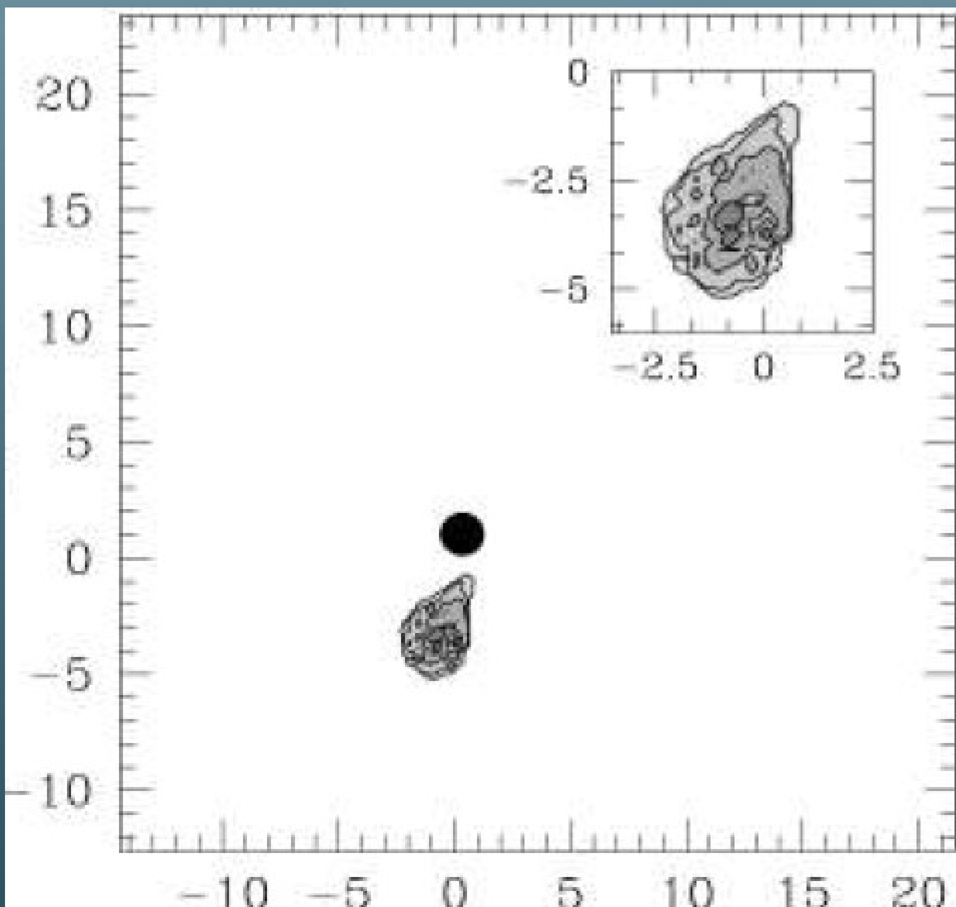
Lorenz gauge condition:

$$\nabla_{\mu} A^{\mu} = 0 \quad \Rightarrow \quad \partial_t \Phi + \nabla \cdot \mathbf{A} = 0$$

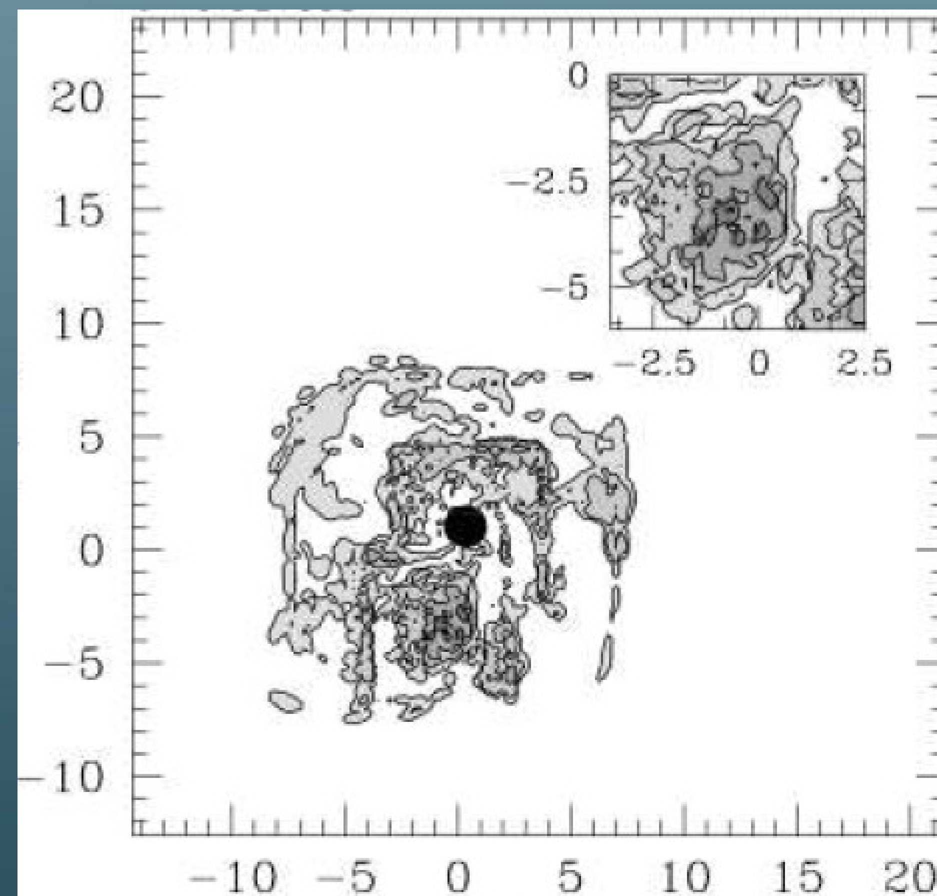
$$\nabla \to \mathbf{k} \quad \Rightarrow \quad \partial_t A_{\mu} = M_{\mu\upsilon} A^{\upsilon}$$

Eigenvalues of $\mathbf{M}$: $\quad \lambda_{1,2} = \pm 1, \; \lambda_3 = \lambda_4 = \mathbf{v} \cdot \mathbf{k}$

EM Gauge Comparison: Magnetic Energy Density



Lorenz EM gauge



Constant Phi EM gauge